Math 4300 - Homework # 4

Betweenness

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1. In the Euclidean plane, let $A = (-1, -2)$, $B = (2, 1)$, and $C = (0, -1)$.
   (a) Determine if A, B, C are collinear or not. Draw a picture.
   (b) If the points are collinear, Determine if $A - B - C$, $A - C - B$, or $B - A - C$.
   (c) Determine if $B - C - A$.

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2. In the hyperbolic plane, let $A = (1, 2)$, $B = (3, 4)$ and $C = (4, \sqrt{19})$.
   (a) Determine if A, B, C are collinear or not. Draw a picture.
   (b) If the points are collinear, Determine if $A - B - C$, $A - C - B$, or $B - A - C$.
   (c) Determine if $B - C - A$.

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3. In the hyperbolic plane, let $A = (1, 2)$, $B = (1, 4)$ and $C = (1, 5)$.
   (a) Determine if A, B, C are collinear or not. Draw a picture.
   (b) If the points are collinear, Determine if $A - B - C$, $A - C - B$, or $B - A - C$.

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4. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let $A, B \in \mathscr{P}$ with $A \neq B$.
   Let $C \in \overleftrightarrow{AB}$. Prove that one and only one of the following can be true: $C - A - B$ or $C = A$ or $A - C - B$ or $C = B$ or $A - B - C$.

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5. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let ℓ be a line and A, B, C be distinct points on ℓ. Prove that either $A - B - C$ or $A - C - B$ or $B - A - C$.

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6. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A, B, C, D be points from $\mathscr{P}$. Prove that if $A-B-C$ and $B-C-D$, then $A-B-D$ and $A-C-D$.

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7. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A, B, C, D be points from $\mathscr{P}$. Assume that $D \neq B$. Prove that if $A-C-D$ and $A-C-B$, then $A-D-B$ or $A-B-D$.

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8. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A, B, C, D be points from $\mathscr{P}$. Prove that if $A-D-C$ and $A-C-B$, then $A-D-B$.

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9. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A, B, P, Q be points from $\mathscr{P}$. Prove that if $A-Q-B$ and $A-P-B$ and $P-C-Q$, then $A-C-B$.

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10. Consider the Euclidean plane $\mathscr{E} = (\mathbb{R}^2, \mathscr{L}_E, d_E)$. Let $A, B, C \in \mathbb{R}^2$ be distinct points. Prove that $A-B-C$ if and only if there exists a real number t with $0 < t < 1$ and $B = A + t(C-A)$.
